

# On Partially Massless Bimetric Gravity

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**ABSTRACT:** We extend the notion of the Higuchi bound and partial masslessness to ghost-free nonlinear bimetric theories. This can be achieved in a simple way by first considering linear massive spin-2 perturbations around maximally symmetric background solutions, for which the linear gauge symmetry at the Higuchi bound is easily identified. Then, requiring consistency between an appropriate subset of these transformations and the dynamical nature of the backgrounds, fixes all but one parameter in the bimetric interaction potential. This specifies the theory up to the value of the Fierz-Pauli mass and leads to the unique candidate for nonlinear partially massless bimetric theory.

**KEYWORDS:** modified gravity, massive gravity, higher spin fields

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## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>The Higuchi bound and partial masslessness in linear massive gravity</b>	<b>3</b>
<b>3</b>	<b>De Sitter solutions and their perturbations in bimetric theory</b>	<b>3</b>
<b>4</b>	<b>Determination of the partially massless bimetric theory</b>	<b>5</b>
<b>5</b>	<b>Nonlinear scaling symmetry</b>	<b>7</b>
<b>6</b>	<b>Discussion</b>	<b>8</b>

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## 1 Introduction

Fierz and Pauli (FP) obtained a ghost-free linear theory of massive spin-2 fields in flat space [1, 2]. When considered in de Sitter backgrounds, the theory has interesting features associated with the Higuchi bound,  $m_{\text{FP}}^2 = \frac{2}{3}\Lambda$ , that determines the mass of the spin-2 state in terms of the cosmological constant [3, 4]. A new gauge symmetry appears at the bound and eliminates the longitudinal mode of the spin-2 field, leaving behind only 4 propagating modes [5–11]. The resulting theory is the linear partially massless gravity on a de Sitter background.

To better understand the origin and consequences of this symmetry, one needs to work with a fully dynamical, nonlinear version of the FP theory. On general grounds, such nonlinear massive spin-2 theories require working with two metrics, say,  $g_{\mu\nu}$  and  $f_{\mu\nu}$ . But they are also generically plagued by the Boulware-Deser ghost instability [12, 13]. A major breakthrough in this field was the work of [14, 15]. Here, the authors developed the nonlinear massive gravity for a flat reference metric  $f_{\mu\nu} = \eta_{\mu\nu}$ , the dRGT model, and established that it was ghost-free in a certain “decoupling limit” analysis [16, 17] that proved to be very powerful for the purpose. All subsequent developments in the field are based on this breakthrough. However, the decoupling limit analysis cannot show the absence of ghost away from this specific limit. That the model remained ghost-free at the complete nonlinear level was proven in [18].

As such the dRGT model does not admit de Sitter solutions since it was established for  $f_{\mu\nu} = \eta_{\mu\nu}$ , to which the decoupling limit analysis is mostly confined. However, the nonlinear analysis of [18] could be used to prove that massive gravity with generic non-dynamical reference metric  $f_{\mu\nu}$  [19] was ghost-free at the complete nonlinear level [20]. Finally the bimetric theory was obtained and proven to be ghost-free in [21, 22]. This theory admits completely dynamical de Sitter solutions. Hence one hopes that it provides the natural setup for investigating the issue of partial masslessness and the associated symmetries.

It is now interesting to ask whether there exists a nonlinear extension of the linear partially massless FP gravity within the family of ghost-free massive gravity and bimetric theories. In the context of massive gravity, this question was recently investigated in [23], in a decoupling limit specifically developed for the de Sitter space. The authors discover the parameter values for which the Stückelberg field that captures the dynamics of the helicity-zero mode of the graviton is removed to all orders in the decoupling limit. If this result extends beyond the decoupling limit, then the theory has a nonlinear gauge symmetry that removes the helicity-zero excitation, and thus constitutes the nonlinear partially massless theory of gravity. For a related investigation, see [24].

Here we investigate the problem in the ghost-free bimetric setup where the background are not fixed but arise dynamically. It is shown that by demanding consistency between the dynamical backgrounds and just a subset of the linear gauge transformations known from FP theory, one can arrive at the unique partially massless bimetric theory.

Other recent works on partial masslessness but in somewhat different setups include [25], as well as [26] in the “new massive gravity” framework of [27, 28]. The issue arises also in the context of higher-spin theories (for a review, see [29]).

## 2 The Higuchi bound and partial masslessness in linear massive gravity

The linear Fierz-Pauli equation for a massive graviton on a fixed de Sitter background reads,

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma} - \Lambda \left( h_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} h_{\rho\sigma} \right) + \frac{m_{\text{FP}}^2}{2} (h_{\mu\nu} - \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} h_{\rho\sigma}) = 0. \quad (2.1)$$

$\Lambda$  is the cosmological constant and  $m_{\text{FP}}$  is the mass of the spin-2 fluctuation.  $\bar{g}_{\mu\nu}$  is the de Sitter metric and the kinetic operator is given by,

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} h_{\rho\sigma} = -\frac{1}{2} \left[ \delta_\mu^\rho \delta_\nu^\sigma \bar{\nabla}^2 + \bar{g}^{\rho\sigma} \bar{\nabla}_\mu \bar{\nabla}_\nu - \delta_\mu^\rho \bar{\nabla}^\sigma \bar{\nabla}_\nu - \delta_\nu^\rho \bar{\nabla}^\sigma \bar{\nabla}_\mu - \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \bar{\nabla}^2 + \bar{g}_{\mu\nu} \bar{\nabla}^\rho \bar{\nabla}^\sigma \right] h_{\rho\sigma}. \quad (2.2)$$

The mass term breaks the symmetry under infinitesimal reparameterizations.

It is known that, in this theory, a curious role is played by the Higuchi bound,

$$m_{\text{FP}}^2 = \frac{2}{3} \Lambda. \quad (2.3)$$

Above the bound,  $m_{\text{FP}}^2 > \frac{2}{3} \Lambda_g$ , (2.1) propagates only the five healthy polarizations of the massive spin-2 fluctuation. In fact, the mass term is fixed uniquely by demanding that a sixth ghost mode decouples [1, 2]. Below the bound,  $m_{\text{FP}}^2 < \frac{2}{3} \Lambda_g$ , the helicity zero component of the spin-2 field becomes a ghost and the theory becomes unstable [3, 4]. But precisely at the Higuchi bound, (2.1) develops a new gauge symmetry that decouples the helicity zero component, leaving only four healthy propagating modes [5–10]. The theory with this value for the mass is often referred to as *partially massless*.

The new linear gauge symmetry that emerges at the Higuchi bound reads [5],

$$h_{\mu\nu} \longrightarrow h_{\mu\nu} + \delta h_{\mu\nu} \quad \text{with} \quad \delta h_{\mu\nu} \equiv \left( \nabla_\mu \nabla_\nu + \frac{\Lambda}{3} \bar{g}_{\mu\nu} \right) \xi(x), \quad (2.4)$$

where,  $\xi(x)$  is an arbitrary gauge transformation parameter. Note that the solutions of  $\delta h_{\mu\nu} = 0$  give conformal Killing transformations  $\delta x^\mu = \nabla^\mu \xi$  on de Sitter space. These are excluded since the FP theory has no coordinate invariance.

Understanding the origin of (2.4) and partial masslessness requires a nonlinear version of the FP theory that, furthermore, treats the background dynamically. This is the ghost-free bimetric theory that will be reviewed in the next section and which will be shown to provide a natural setup for addressing the issue of partial masslessness.

## 3 De Sitter solutions and their perturbations in bimetric theory

The most general bimetric action with the correct combination of the kinetic and potential terms that avoids the Boulware-Deser ghost at the nonlinear level is given by, [21, 22]

$$S_{gf} = \int d^4x \left[ m_g^2 \sqrt{g} R(g) + m_f^2 \sqrt{f} R(f) + m^4 \sqrt{g} \sum_{n=0}^4 \beta_n e_n \left( \sqrt{g^{-1}f} \right) \right]. \quad (3.1)$$

The  $e_n(S)$  are the elementary symmetric polynomials of the eigenvalues of the matrix  $S$ . The action has seven independent parameter, the Planck masses  $m_g$  and  $m_f$ , and the five dimensionless  $\beta_n$ . The mass scale  $m$  is degenerate with these. The potential is an extension of the massive gravity case in [15] in the formulation of [19].

The equations of motion for  $g_{\mu\nu}$  and  $f_{\mu\nu}$ , obtained from (3.1) are given, for example, in [30] on which the present section is based. To address the Higuchi bound, we are interested in de Sitter solutions. Generic de Sitter solutions of the sourceless bimetric equations are of the type <sup>1</sup>,

$$\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu} \quad (3.2)$$

where  $c$  is a constant, generically determined by the seven parameters of the theory. Indeed, for this ansatz, the  $g$  and  $f$  equations of motion reduce to two copies Einstein's equation,

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2} \bar{g}_{\mu\nu} R(\bar{g}) - \Lambda_g \bar{g}_{\mu\nu} = 0, \quad (3.3)$$

and a similar equation, again for  $\bar{g}_{\mu\nu}$ , but with a cosmological constant  $\Lambda_f$ , where,

$$\Lambda_g = \frac{m^4}{m_g^2} (\beta_0 + 3c\beta_1 + 3c^2\beta_2 + c^3\beta_3), \quad \Lambda_f = \frac{m^4}{c^2 m_f^2} (c\beta_1 + 3c^2\beta_2 + 3c^3\beta_3 + c^4\beta_4). \quad (3.4)$$

The consistency of the two equations then requires,

$$\Lambda_g = \Lambda_f. \quad (3.5)$$

This provides, in general, a quartic equation that determines  $c$  in terms of the 6 combinations of the seven parameters of the theory,

$$c = c(\alpha, \beta_n), \quad \text{with,} \quad \alpha \equiv \frac{m_f}{m_g}. \quad (3.6)$$

For the purpose of these solutions, the relevant regions of the parameter space are those that lead to a real  $c$  and a real, positive Fierz-Pauli mass, as given below, for the fluctuation.

Let us now consider linear perturbations around this background,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{m_g} \delta g_{\mu\nu}, \quad f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \frac{1}{m_g} \delta f_{\mu\nu}. \quad (3.7)$$

The equations of motion for the perturbations can be diagonalized in terms of a massless mode  $\delta G_{\mu\nu}$  and a massive mode  $\delta M_{\mu\nu}$ ,

$$\delta G_{\mu\nu} = \frac{\delta g_{\mu\nu}}{m_g} + \alpha^2 \frac{\delta f_{\mu\nu}}{m_f}, \quad \delta M_{\mu\nu} = \frac{1 + \alpha^2 c^2}{2c} \left( \frac{\delta f_{\mu\nu}}{m_f} - c^2 \frac{\delta g_{\mu\nu}}{m_g} \right). \quad (3.8)$$

The normalizations are explained below. They satisfy the corresponding equations,

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta G_{\rho\sigma} - \Lambda_g \left( \delta G_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \delta G_{\rho\sigma} \right) = 0, \quad (3.9)$$

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} \delta M_{\rho\sigma} - \Lambda_g \left( \delta M_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \delta M_{\rho\sigma} \right) + \frac{1}{2} m_{\text{FP}}^2 (\delta M_{\mu\nu} - \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \delta M_{\rho\sigma}) = 0, \quad (3.10)$$

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<sup>1</sup>Simply requiring  $g_{\mu\nu}$  to be a dS spacetime for generic  $\beta_n$  restricts the solutions to such proportional backgrounds.

where  $\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma}$  is given by (2.2) and the Fierz-Pauli mass of the massive spin-2 mode reads,

$$m_{\text{FP}}^2 = \frac{m^4}{m_g^2} (1 + \alpha^{-2} c^{-2}) (c\beta_1 + 2c^2\beta_2 + c^3\beta_3) . \quad (3.11)$$

The normalizations in (3.8) are chosen for convenience so that the mass eigenstates can be regarded as fluctuation of nonlinear modes [30],

$$G_{\mu\nu} = g_{\mu\nu} + \alpha^2 f_{\mu\nu} , \quad M_{\mu\nu} = G_{\mu\rho} (\sqrt{g^{-1}} f)^\rho{}_\nu - c G_{\mu\nu} . \quad (3.12)$$

Equation (3.10) for the massive spin-2 fluctuation coincides with the Fierz-Pauli equation (2.1). In particular, at the Higuchi bound,  $m_{\text{FP}}^2 = \frac{2}{3}\Lambda_g$ , it has the same extra gauge invariance (2.4). Now, taking the presence of the massless mode  $\delta G_{\mu\nu}$  also into account, the corresponding symmetry transformations in the bimetric theory become,

$$\delta M_{\mu\nu} \rightarrow \delta M_{\mu\nu} + \left( \nabla_\mu \nabla_\nu + \frac{\Lambda}{3} \bar{g}_{\mu\nu} \right) \xi(x) , \quad \delta G_{\mu\nu} \rightarrow \delta G_{\mu\nu} . \quad (3.13)$$

While, superficially, this is very similar to Fierz-Pauli massive gravity, a major difference is that in the bimetric case the background is dynamical and is not fixed by hand. Demanding compatibility between a subset of (3.13) and the dynamical nature of the background is powerful enough to uniquely determine the partially massless nonlinear bimetric theory. This is explained below.

#### 4 Determination of the partially massless bimetric theory

From (3.13) and (3.8), one can easily read off the transformations of  $\delta g_{\mu\nu}$  and  $\delta f_{\mu\nu}$ ,

$$\delta g_{\mu\nu} \rightarrow \delta g_{\mu\nu} + a \left( \nabla_\mu \nabla_\nu + \frac{\Lambda}{3} \bar{g}_{\mu\nu} \right) \xi(x) , \quad \delta f_{\mu\nu} \rightarrow \delta f_{\mu\nu} + b \left( \nabla_\mu \nabla_\nu + \frac{\Lambda}{3} \bar{g}_{\mu\nu} \right) \xi(x) , \quad (4.1)$$

where the constants  $a$  and  $b$  are given in terms of  $\alpha$  and  $c$ . The gauge transformations, being symmetries, are trivial solutions of the linearized equations of motion.

The crucial point to note is that, for a dynamical field, say  $g_{\mu\nu}$ , the split into a background part  $\bar{g}_{\mu\nu}$  and a fluctuation  $\delta g_{\mu\nu}$  is not unique since, in principle, infinitesimal symmetry transformations can be shifted between the two. Hence, one can always transfer a part of  $\delta g_{\mu\nu}$  to the background  $\bar{g}_{\mu\nu}$  to get,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} = \bar{g}'_{\mu\nu} + \delta g'_{\mu\nu} , \quad (4.2)$$

where the backgrounds  $\bar{g}_{\mu\nu}$  and  $\bar{g}'_{\mu\nu}$  differ by an infinitesimal symmetry transformation.

In bimetric theory, the two metrics are dynamical and come with their own equations of motion. If we assume that the theory has a symmetry that manifests itself as (3.13) around the backgrounds considered, then this argument tells us that it should be possible to transfer the infinitesimal transformations (4.1) from the fluctuations to the backgrounds  $\bar{g}_{\mu\nu}$  and  $\bar{f}_{\mu\nu}$  and end up with new consistent background solutions. Now, it is easy to verify that for a

general  $\xi(x)$  in (4.1), the new backgrounds  $\bar{g}'_{\mu\nu}$  and  $\bar{f}'_{\mu\nu}$  cannot not satisfy  $f' = c'^2 g'$ , with a constant  $c'$ . Hence this operation will take us out of the general class of backgrounds for which the Higuchi bound and the extra symmetry transformation are known.

However, the subset of transformations (4.1) with constant  $\xi$  is consistent with the proportional background ansatz and can be used for this purpose. In this case,

$$\delta g_{\mu\nu} \rightarrow \delta g_{\mu\nu} + a \frac{\Lambda}{3} \xi \bar{g}_{\mu\nu}, \quad \delta f_{\mu\nu} \rightarrow \delta f_{\mu\nu} + b \frac{\Lambda}{3} \xi \bar{g}_{\mu\nu}. \quad (4.3)$$

Transferring these from the fluctuations to the backgrounds gives,

$$\bar{g}'_{\mu\nu} = \bar{g}_{\mu\nu} + a \frac{\Lambda}{3} \xi \bar{g}_{\mu\nu}, \quad \bar{f}'_{\mu\nu} = \bar{f}_{\mu\nu} + b \frac{\Lambda}{3} \xi \bar{g}_{\mu\nu} \quad (4.4)$$

Now, it is obvious that we get  $f' = c'^2 g'$  with a constant  $c'(\xi)$ . However,  $c' \neq c$ . On the other hand, from the previous section we know that generically, for proportional ansatz, the bimetric equations determine  $c$  in terms of the parameters of the theory and there is no room for varying it.

The implication of course is that  $\bar{g}'_{\mu\nu}$  and  $\bar{f}'_{\mu\nu}$  are not valid background solutions. This can only happen if the transformations that generate them do not correspond to symmetries of the nonlinear theory. Thus the only parameter values for which the bimetric theory is consistent with the transformations (4.3), are those for which the equation for  $c$  (3.5) does not determine  $c$  at all! This is the necessary condition for the existence of a nonlinear partially massless bimetric theory.

Note that by fixing  $\xi(x)$  to be constant, we restrict ourselves to only part of the gauge group, and if in fact the full gauge symmetry is realized at the nonlinear level, the theory obtained from using only the subgroup will definitely contain the partially massless theory including the full symmetry.

Having established a necessary condition for the existence of the nonlinear partially massless theory, one can easily find the parameter values that leave the  $c$  in the proportional background ansatz undetermined. The equation (3.5) that determines  $c$  can be written as,

$$\beta_1 + (3\beta_2 - \alpha^2\beta_0)c + (3\beta_3 - 3\alpha^2\beta_1)c^2 + (\beta_4 - 3\alpha^2\beta_2)c^3 + \alpha^2\beta_3c^4 = 0. \quad (4.5)$$

Thus the parameter combination for which  $c$  remains undetermined is,

$$\alpha^2\beta_0 = 3\beta_2, \quad 3\alpha^2\beta_2 = \beta_4, \quad \beta_1 = \beta_3 = 0. \quad (4.6)$$

Remarkably, this fixes all but one of the  $\beta_n$  which immediately implies that if there is a nonlinear partially massless theory it has to be this one. Note that with the choice (4.6) we have

$$m_{\text{FP}}^2 = 2 \frac{m^4}{m_g^2} (\alpha^{-2} + c^2) \beta_2 = \frac{2}{3} \Lambda_g, \quad (4.7)$$

so in particular the massive fluctuation has a mass at the Higuchi bound.

As a side remark we note that the action is symmetric under  $\alpha^{-1}g_{\mu\nu} \leftrightarrow \alpha f_{\mu\nu}$  which is a consequence of  $\sqrt{g} e_n(S) = \sqrt{f} e_{4-n}(S^{-1})$  [21]. For any parameters that satisfy  $\alpha^{-n}\beta_n = \alpha^{n-4}\beta_{4-n}$  the symmetry is realized, and the candidate for nonlinear partially masslessness falls into this class.

The reasoning presented here is simple and straightforward enough that it can be easily generalized to any number of dimensions and to theories with multiple spin-2 fields [31, 32].

## 5 Nonlinear scaling symmetry

Taking into account the value for  $\Lambda_g$  given in (4.7), it is easy to see that for the parameter choice (4.6) the background equations (3.3) are invariant under the simultaneous transformations,

$$c \longrightarrow c + a, \quad \bar{g}_{\mu\nu} \longrightarrow \frac{\alpha^{-2} + c^2}{\alpha^{-2} + (c + a)^2} \bar{g}_{\mu\nu}, \quad a \in \mathbb{R}. \quad (5.1)$$

The linearized versions of these read

$$c \longrightarrow c + \delta c, \quad \bar{g}_{\mu\nu} \longrightarrow \bar{g}_{\mu\nu} + \delta \bar{g}_{\mu\nu} \equiv \bar{g}_{\mu\nu} - \frac{2c}{\alpha^{-2} + c^2} \delta c \bar{g}_{\mu\nu}. \quad (5.2)$$

Moreover, using  $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$ , we find that  $\bar{f}_{\mu\nu}$  transforms as

$$\bar{f}_{\mu\nu} \longrightarrow \bar{f}_{\mu\nu} + \delta \bar{f}_{\mu\nu} \equiv \bar{f}_{\mu\nu} + 2c\delta c \bar{g}_{\mu\nu} + c^2 \delta \bar{g}_{\mu\nu}. \quad (5.3)$$

We will now reverse the arguments that were given in section 4 and translate these scalings of the backgrounds into transformations of the fluctuations in order to see if we can re-arrive at the  $\xi = \text{const.}$  version of the transformation (3.13) for the massive fluctuation in de Sitter space. For this we identify  $\delta f_{\mu\nu} = m_f \delta \bar{f}_{\mu\nu}$  and  $\delta g_{\mu\nu} = m_g \delta \bar{g}_{\mu\nu}$ . Then we see that the massless and massive fluctuation transform as

$$\delta G_{\mu\nu} \longrightarrow \delta G_{\mu\nu}, \quad \delta M_{\mu\nu} \longrightarrow \delta M_{\mu\nu} + \frac{\Lambda_g}{3} \frac{m_f^2}{\beta_2 m^4} \bar{g}_{\mu\nu} \delta c, \quad (5.4)$$

where we have used (4.7) to express the transformation of  $\delta M_{\mu\nu}$  in terms of  $\Lambda_g$ . As required, the symmetry transformation (5.2) leaves the massless fluctuation invariant<sup>2</sup> while the massive fluctuation transforms as under a scaling. The transformation of  $\delta M_{\mu\nu}$  is precisely of the form (3.13) with  $\xi = \frac{m_f^2}{\beta_2 m^4} \delta c$ . Thus we have reproduced the linearized symmetry with constant gauge parameter known to be present at the Higuchi bound through the constant but nonlinear transformation (5.1).

We emphasize once more that all of these conclusions become invalid once we choose parameters different from (4.6) since then the background equations do determine  $c$  and there is no invariance at the background level. Thus the unique candidate for a nonlinear partially massless theory is the one specified by (4.6).

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<sup>2</sup>Note that, in addition, the background  $\bar{G}_{\mu\nu} = (1 + \alpha^2 c^2) \bar{g}_{\mu\nu}$  of the nonlinear massless field  $G_{\mu\nu} = g_{\mu\nu} + \alpha^2 f_{\mu\nu}$  is invariant under the full nonlinear scaling symmetry (5.1).

## 6 Discussion

In section 4 we argued that the bimetric parameter space that we derive, by focusing on constant gauge parameters in the symmetry transformation (3.13), must contain the nonlinear partially massless theory with the full symmetry. Since the interaction potential of the theory specified by (4.6) has only one remaining free parameter, we conclude that the nonlinear partially massless theory has to be exactly the one we derived here. In any ghost-free bimetric theory the Fierz-Pauli mass can be placed on the Higuchi bound by fixing one of the  $\beta_n$  in terms of the others. But partial masslessness at the linear level extends to the nonlinear theory only for the special value of parameters found here.

To compare with the results of [23], one has to take the massive gravity limit,  $m_f \rightarrow \infty$ . Although the non-dynamical backgrounds considered there cannot be derived in a consistent way from our bimetric equations (3.3), the findings of [23] support the absence of the helicity-zero mode around maximally symmetric backgrounds at the nonlinear level. This strongly suggests that the linear gauge symmetry (3.13) in fact has a nonlinear generalization and removes one degree of freedom in the full theory. In contrast, in this work we have obtained the nonlinear version of this symmetry but for constant gauge parameter, which may be of help in identifying the full nonlinear local symmetry.

For the (non-proportional) homogeneous and isotropic background solutions found in [33], the parameter choice (4.6) precisely removes the equation that determines the ratio of the scale factors of  $g$  and  $f$ . We therefore expect a gauge symmetry to be present for these backgrounds as well. Determining the exact form of this symmetry can be of further help in identifying its nonlinear extension. We leave the investigation of these interesting possibilities for future work. Models with  $\beta_1 = \beta_3 = 0$  also give homogeneous and isotropic background solutions that are degenerate with general relativity [33]. This makes their phenomenology interesting also from a cosmological point of view. It is also interesting to understand the partially massless bimetric theory in connection with the self-protection mechanism discussed in [34–37].

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